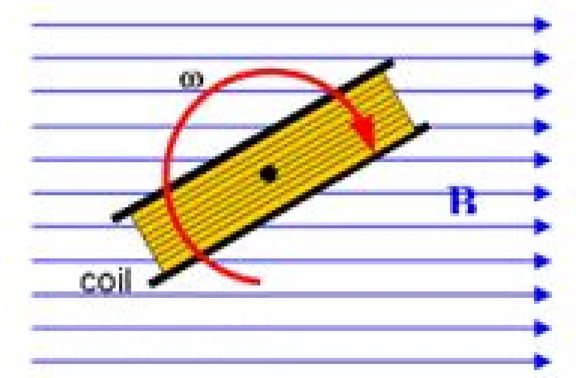
WJEC (Wales) Physics A-level

Topic 4.A: Alternating Currents

Notes

Electromagnetic Induction (Rotating Coil in a Field)



https://spark.iop.org/sites/default/files/image/plane-of-coil-at-an-angle-to-the-magnetic-field.gif

A square coil of A cross-sectional area and N turns is **rotating with angular velocity** ω in a magnetic field B. The flux of each loop is:

Flux = $BA\ cos\theta$, where θ is the angle between the magnetic field and the normal of the coil.

For N loops: **Flux linkage** = $N = BAN\ cos\theta$

We can say that the coil starts with θ=0, therefore: θ=ωt

We can therefore rewrite the previous expression: **Flux linkage** = $N = BAN\ cos\omega t$

Using **Faraday's law**, the induced emf can be calculated:

$$V = -\frac{\Delta(N\Phi)}{\Delta t} = -\frac{\Delta(BAN\cos\omega t)}{\Delta t}$$

Which is: $V = \omega BAN \sin\omega t$

Period, Frequency and RMS Values

We can describe the nature of **alternating currents** in the same way as we can describe **waves**: in terms of their **frequency**, **time period**, and **peak value** or **amplitude**. An alternating current differs from a direct current in that it reverses direction periodically. Oscillating like a wave, it reaches a peak value in each direction whereas a direct current stays at a constant positive value.

The **time period** is how long does it take for one cycle to be completed.
The **frequency** is how many peaks happen per second.
The **peak value** is the maximum value of the I or V.

Alternating currents typically vary **sinusoidally**, and therefore their current or voltage can be represented by an equation of the form:

$$x = x_0 sin(\omega t)$$

where x is the voltage or amplitude, x_0 is its peak value magnitude, t is the time and ω is the angular frequency of the wave.
The angular frequency is related to the normal frequency f by $\omega = 2\pi f$.

The **mean power** $\langle P \rangle$ delivered by the alternating current wave is given as $\langle P \rangle = \langle I_0{}^2 R\, sin^2(\omega t) \rangle$. Since $I_0{}^2$ and R are constant, and the average of $sin^2(\omega t)$ is ½, the mean power is equal to **half the maximum power** $\langle P \rangle = \frac{1}{2}P = \frac{1}{2}I_0{}^2 R = \frac{1}{2}V_0{}^2/R$. In the same way we can average the squares of the voltage or current by comparing with their maximum values: $\langle V^2 \rangle = \frac{1}{2}V_0{}^2$, $\langle I^2 \rangle = \frac{1}{2}I_0{}^2$.

The **root mean square (rms)** voltage or current is defined as the square root of the mean of the squares of the voltage/current stated above. For a sinusoidal alternating current, this yields rms values of

$$V_{rms} = \frac{V_0}{\sqrt{2}}, \quad I_{rms} = \frac{I_0}{\sqrt{2}}$$

The rms value gives the value of direct current that would give the same heating effect as the alternating current on the same resistor.

It is important to note that $V_0 = \omega BAN$ (because when t=0, sin($\omega \times 0$) = 1), and hence:

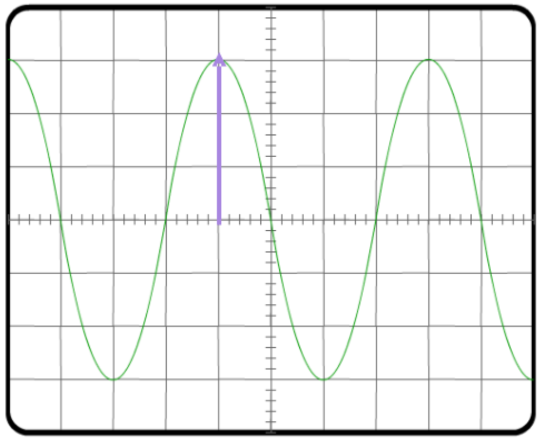
$$V_{rms} = \frac{\omega BAN}{\sqrt{2}}$$

The Oscilloscope

An oscilloscope is a device that allows us to see how **voltage** changes with **time**. The y axis is voltage, and the x axis is time.

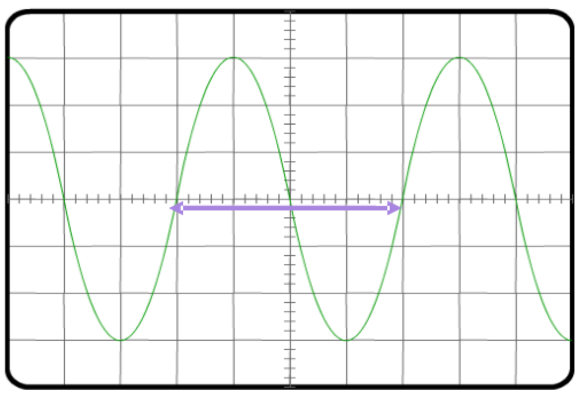
For **direct current**, the voltage does not vary, and therefore the oscilloscope will only show a **straight horizontal line**, the height of which on the y axis corresponds to the value of V.

For **alternating current**, the voltage appears as a **sinusoidal wave** (as mentioned before):



In the picture above, the purple arrow represents the maximum value of V (the peak value of V).

To measure the frequency, a full period has to be measured, and remembering that $f = 1/T$, where T is the period, the frequency can be obtained.



From the settings of the oscilloscope it can be determined how many V per division in the y axis, and seconds per division for the x axis are shown.

RLC Circuits

RLC circuits are a.c. circuits that have a **resistor** (with resistance R), a **capacitor** (with capacitance C) and an **inductor** (with inductance L). These elements can be combined in parallel or in series, just as with a d.c. circuit, but we will only see series circuits.

The current in an a.c. current is given by $I = I_0 \sin \omega t$

Inductors are usually long wires turned into many loops, making a coil. When an electric current passes through an inductor, the inductor stores energy in a magnetic field B. Because we are only considering series **RLC circuits**, there is only **one current across all the circuit**, it does not vary. For an inductor, the current lags 90º behind the potential difference. This can be seen when from the expression of the **instantaneous potential difference** in the inductor:

$$V_L = I_0 X_L \cos \omega t$$

There is a 90º difference between sin and cos. The term X_L is called **reactance for an inductor**, and it can be calculated with:

$$X_L = 2\pi f L \quad \text{or} \quad = \omega L$$

For resistors, the **potential difference** is always in phase with the current:

$$V_R = IR = I_0 R \sin \omega t$$

Similarly, the expression for the **capacitor** is:

$$V_C = -I_0 X_C \cos \omega t$$

Which means that the potential difference lags 90º behind the current. **X_C is the reactance for a capacitor:**

$$X_C = \frac{1}{\omega C}$$

We can see that the potential differences in the capacitor and the inductor always oppose each other, and thus are always in antiphase.

With the resistance and the two reactances, a value called **impedance** can be calculated. The **impedance** (Z) is the measure of the opposition that a circuit presents to the current when a voltage is applied. If the circuit only has a resistor, the **impedance** will be the value of resistance for that resistor. Because the reactance and the resistance are always at 90º with respect to each other, we used Pythagoras' theorem to combine them:

$$Z = \sqrt{(X_L - X_C)^2 + R^2}$$

Z can also be calculated as such: $Z = \frac{V_{rms}}{I_{rms}}$

This is similar to dc circuits, where R = V/I (Z is is similar to R in the traditional d.c. resistor circuits) or V = R x I.

This can be also applied to each individual component of the circuit:

$V_R = I_{rms}R$ for the **resistor**

$V_L = I_{rms}X_L$ for the **inductor**

$V_C = I_{rms}X_C$ for the **capacitor**

Because there is only one current in the circuit, it is also useful to remember that:

$$I_{rms} = \frac{V_R}{R} = \frac{V_L}{X_L} = \frac{V_C}{X_C} = \frac{V_S}{Z}$$

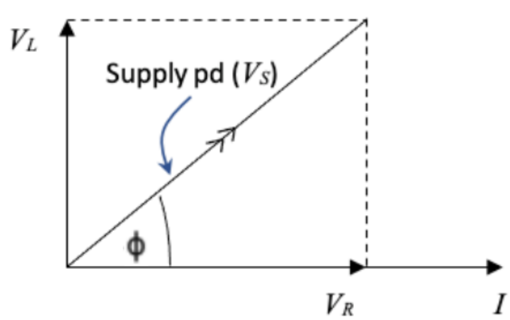
An important aspect of RLC circuits is the dissipated **power**, specifically in capacitors and inductors. Remember that Power = Current x Voltage, so if the current or the voltage is negative there will be a negative power flow, which means power will be flowing out of the capacitor/inductor. Over one cycle, there is as much positive as negative power, which means that when all of the contributions are added up the **average dissipated power** is 0.

Phasors

Phasors are diagrams that allow us to visualise the differences in **phase angle** between the different voltages in a circuit. In a phasor diagram, the voltages are represented as arrows (even though they are scalars), and the intensity is also there to show what the source current is (remember that the angle is always between the source current/voltage and the current or voltage in each component).

Phasors can be used to add potential differences across RC, RL and RLC circuits. A RC circuit can be thought of as a RLC circuit with L=0. Similarly, a RL circuit is just a RLC circuit with C=0.

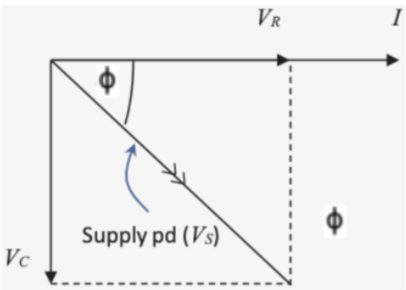
The phasor diagram for a **RL circuit** is:



https://d3kp6tphcrvm0s.cloudfront.net/el20-21_3-6/4/5

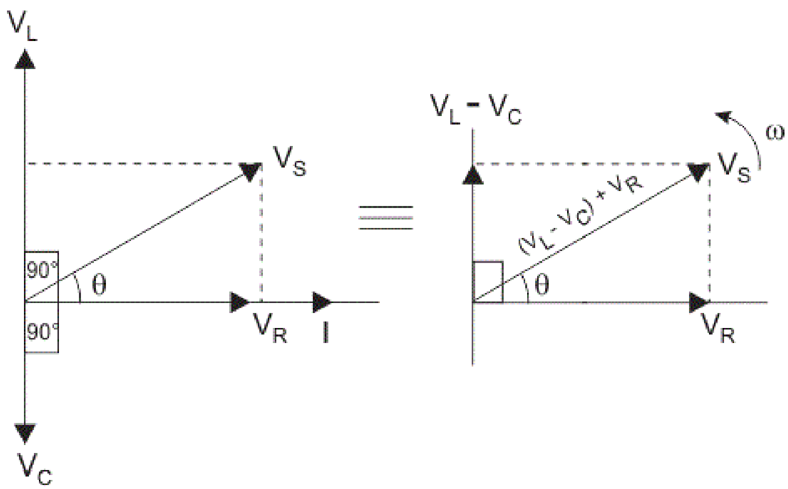
Where **ϕ** is defined as: $\tan\phi = \frac{X_L - X_C}{R}$

For a **RC circuit**:



In this case, because there is no X_L, the angle is negative (or clockwise).

Finally, for a **RLC circuit**:



https://www.electrical4u.com/electrical/wp-content/uploads/2013/06/vector-diagram-of-rlc-circuit.gif

In this figure, θ is used, but in our notes it is equivalent to ϕ.

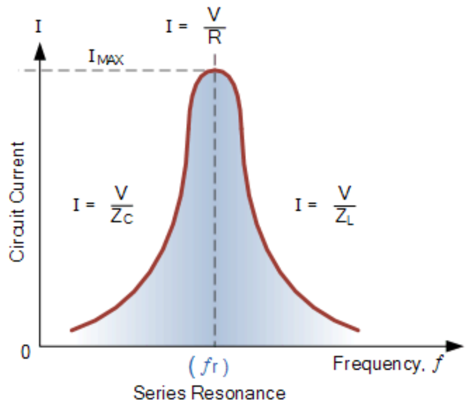
We can see that in all three cases the source voltage is calculated by adding the voltages across the different components as vectors with Pythagoras' theorem, because they are always at a 90° angle. Note that in the last figure, the source voltage, V_S, is the **vector addition** of the other voltages, meaning that to calculate it numerically, the formula is:

$$V_S = \sqrt{V_R^2 + (V_L - V_C)^2}$$

Phasor diagrams can also be drawn for the resistance and the reactances, and by adding them using Pythagoras' theorem the impedance Z is obtained.

Resonance

In an RLC circuit, as the frequency of the supply voltage is changed, this graph can be obtained:



https://www.electronics-tutorials.ws/accircuits/series-resonance.html

The peak in the middle corresponds to the **resonance frequency**, which is the frequency at which X_L and X_C are equal, and therefore, in this formula $Z = \sqrt{(X_L - X_C)^2 + R^2}$, they cancel each other out, leaving Z=R. Both reactances are functions of angular frequency, and thus an expression for the **resonant frequency** can be found:

$X_L = X_C$

$\omega L = \frac{1}{\omega C}$

And therefore, the **resonant frequency** is:

$$\omega = \frac{1}{\sqrt{LC}}$$

In rad/s, as it is angular frequency. It can be changed to Hz using $\omega = 2\pi f$

From the graph above, the **Q factor** determines the **sharpness of the curve**. The lower the value, the broader the curve. The **Q factor** can be calculated with:

$$Q = \frac{V_L}{V_R} \text{ or } \frac{V_C}{V_R}$$